PARAMETRIC PULSE EXCITATION IN DISTRIBUTED MECHANICAL SYSTEMS WITH NONSTATIONARY BOUNDARIES

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In a distributed system whose parameters vary with time the natural oscillation modes are interconnected and so it is possible to get parametric excitation of several synchronized harmonic modes simultaneously. If the natural oscillation spectrum of such a system consists of almost equally spaced lines, then a periodic change of the parameters with time can lead to the excitation of pulse-type oscillations [1]. This phenomenon can occur both in systems whose size varies with time and in systems whose boundary properties are nonstationary. The present paper is devoted to a study of the instability in these systems.

§1. The effect of moving boundaries on the nature of wave phenomena in one-dimensional mechanical systems was considered as early as in [2, 3]. Nikolai [3] was the first to obtain an exact solution to the problem of oscillations in a system whose size varied uniformly with time. Interest in a detailed study of these phenomena has only appeared comparatively recently in connection with the increased operational speeds of machines* which use this type of system as their main elements.

Many publications have now appeared (see, for example, [4, 5]) on the subject of nonresonant phenomena in systems with moving boundaries. On the other hand, almost no attention has been given to the resonance phenomena which result in parametric excitation although it is known that [1] the excited oscillations will be in the form of pulses.

We consider the mechanical system consisting of a stretched string moving with a constant velocity v through two rings which are undergoing harmonic oscillations. We assume that the diameters of the rings are equal to the diameter of the string.

The transverse displacement of the string u satisfies the equation

$$\partial^2 u/\partial t^2 - 2v\partial^2 u/\partial t\partial x - (c^2 - v^2)\partial^2 u/\partial x^2 = 0, \tag{1.1}$$

and the homogeneous boundary conditions

$$u|_{x=a(t)} = u|_{x=b(t)} = 0,$$

where c is the velocity of waves on a stationary string; $a(t) = \lambda \sin \Omega t$; $b(t) = l_0 + \lambda \sin \Omega t$; and λ, Ω, and l_0 are constants.

When stated in this form the problem only remains valid when the traveling waves on the string are reflected from the boundaries, i.e., when†

$$|\lambda\Omega| < c - |v|. \tag{1.2}$$

*We refer here to looms, winding machines, and shaft hoists.

†The problem of the correct form of stating the problem for the dynamics of a variable-length string is discussed in some detail in [1].

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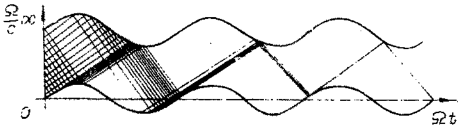

Fig. 1

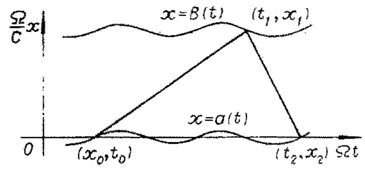

Fig. 2

Following the method proposed in [6], we can arbitrarily divide any initial perturbation of the string into individual sections, each of which contains two pulses propagating as indicated by (1.1):

$$x = (c - v)t + C_1, \quad x = -(c + v)t + C_2,$$

where C_1 and C_2 are appropriate constants.

The individual pulses in the initial perturbations will eventually, after multiple reflections from the boundaries, form into a single pulse (Fig. 1). This interacts with the boundary only at those instants of time when the boundary is moving toward it. Its energy on reflection therefore increases in porportion to the compression of the wave profile as dictated by the double Doppler effect. In the limit $t \to \infty$, the length of the perturbation tends to zero and its energy to infinity (this is because nonlinear effects, such as the way the perturbations influence the movement of the boundary, are not taken into account).

It can be seen from Fig. 1 that the pulse trajectory in the space–time plane $(\Omega x/c, \Omega t)$ is a broken line composed of sections of the characteristics lying between the boundary trajectories. It is natural to suppose that parametric resonance will be possible at least in those cases where the function $f(t)$ which describes the broken line near which the characteristics converge is periodic with a period T which is, of course, a multiple of that of the boundary oscillations.

The problem can therefore be divided into two parts: a) finding the conditions under which a periodic broken line exists; b) finding the conditions for parametric resonance.

§2. We now derive the conditions which have to be imposed on the system parameters to make the function $f(t)$ periodic. The approach we suggested can be used for considering broken lines with any finite T but for the clarity we limit ourselves to the case where the period of $f(t)$ is equal to the time between two successive reflections of the pulse from the same boundary.

Putting $x_2 = x_0$; $t_2 = t_0 + 2\pi N\Omega^{-1}$ (Fig. 1), we get a system of equations which uniquely define the coordinates $x_{0,1}$ and $t_{0,1}$:

$$\begin{cases} x_0 = \lambda \sin \Omega t_0, \\ x_0 = (c - v)\, t_0 + C_1, \\ x_0 = -(c + v)\left(t_0 + 2\pi N\Omega^{-1}\right) + C_2, \\ x_1 = l_0 + \lambda \sin \Omega t_1, \\ x_1 = (c - v)\, t_1 + C_1, \\ x_1 = -(c + v)\, t_1 + C_2. \end{cases}$$

Solving this system for $t_0 + t_1$, we get

$$\cos[(\Omega/2)(t_0 + t_1)] = [\pi N c\Omega^{-1}(1 - v^2/c^2) - l_0]/2\lambda \sin[(\pi N/2)(1 + v/c)]. \tag{2.1}$$

Since t_0 and t_1 are real, Eq. (2.1) only has a solution when

$$\left| \left[\omega\Omega^{-1} N(1 - v^2/c^2) - 1\right]/2\lambda l_0^{-1} \sin[(\pi N/2)(1 + v/c)] \right| \leqslant 1, \tag{2.2}$$

where $\omega = \pi c l_0^{-1}$ is the lowest natural frequency of the corresponding stationary system.

The inequality (2.2) defines the region of the system parameters for which a periodic broken line exists. The boundaries are the surfaces

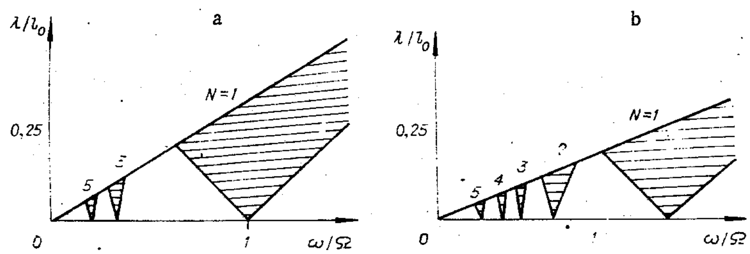

Fig. 3

$$\lambda l_0^{-1} = \left[1 - N\omega\Omega^{-1}(1 - v^2/c^2)\right]/2\left|\sin[(\pi N/2)(1 + v/c)]\right|,$$
$$\lambda l_0^{-1} = \left[N\omega\Omega^{-1}(1 - v^2/c^2) - 1\right]/2\left|\sin[(\pi N/2)(1 + v/c)]\right|.$$

We also have to remember condition (1.2) and so we get one more bounding surface:

$$\pi\lambda/l_0 = \omega\Omega^{-1}(1 - |v|/c).$$

§3. For parametric resonance to occur it is necessary for the characteristics to become concentrated with time. The ratios of the distances between two fairly close characteristics before (ρ_1) and after (ρ_2) reflection from the upper and lower boundaries, respectively, are equal to

$$\sqrt{\frac{1+(1+v/c)^2}{1+(1-v/c)^2}}\cdot\frac{1-v/c-\dot{b}/c}{1+v/c+\dot{b}/c},$$
$$\sqrt{\frac{1+(1-v/c)^2}{1+(1+v/c)^2}}\cdot\frac{1+v/c-\dot{a}/c}{1-v/c-\dot{a}/c}.$$

Since two reflections occur in one period of f(t), the condition for concentration of the characteristics and therefore for parametric resonance can be written

$$\{[1-v/c-\dot{b}(t_1)/c]/[1-v/c+\dot{b}(t_1)/c]\}\cdot\{[1+v/c+\dot{a}(t_2)/c]/[1-v/c-\dot{a}(t_2)/c]\} > 1$$

or

$$\dot{a}(t_2) - \dot{b}(t_1) > 0. \tag{3.1}$$

Substituting into (3.1) the equation for the movement of the boundary (2.2), we get

$$\sin[(\Omega/2)(t_1 + t_2)]\cdot\sin[(\pi N/2)(1 + v/c)] > 0,$$

i.e., when $\sin[(\pi N/2)(1 + v/c)] > 0$, $2k\pi < (\Omega/2)(t_1 + t_2) < (2k + 1)\pi$; when $\sin[(\pi N/2)(1 + v/c)] < 0$, $2(k-1)\pi < (\Omega/2)(t_1 + t_2) < 2k\pi$ ($k = 0, 1, 2, \ldots$); or if $\sin[(\pi N/2)(1 + v/c) \neq 0$, then $\cos[(\Omega/2)(t_1 + t_2]$ can take any values except the extremal values

$$-1 < \cos[(\Omega/2)(t_1 + t_2)] < 1. \tag{3.2}$$

Comparing (3.2) with (2.1), we see that apart from the boundaries and the points where $\sin[(\pi N/2)(1 + v/c)] = 0$, the region where periodic f(t) functions exist is the same as that where we get parametric resonance. Thus parametric resonance is determined by the inequalities

$$\begin{gathered}\left|\left[\omega\Omega^{-1}N(1 - v^2/c^2) - 1\right]/2\lambda l_0^{-1}\sin[(\pi N/2)(1 + v/c)]\right| < 1;\\ \pi\lambda l_0^{-1} < \omega\Omega^{-1}(1 - |v|/c).\end{gathered} \tag{3.3}$$

§4. Conditions (3.3) are quite simple in form. They contain four independent parameters but one of them (N) can take only integral values, to each of which there corresponds a particular zone in the parameter space (ω/Ω, v/c, λ/l_0). The zones have no common points and so the values of N can conveniently be used to number the zones.

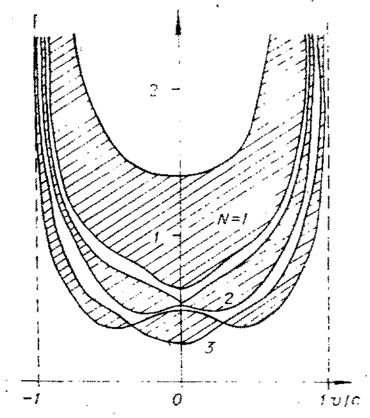

Fig. 4

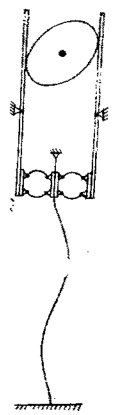

Fig. 5

The present method enables linear losses at boundary reflections to be taken into account quite simply. It is only necessary to multiply the left side of (3.1) by the coefficients Γ_a and Γ_b which characterize the dissipative energy losses occurring on reflection. Using the inequality thus obtained, we can easily find the condition for instability; when v = 0 this becomes

$$\left|\frac{\omega\Omega^{-1}N-1}{2\lambda l_0^{-1}}\right| < \left\{1-\left[\frac{\omega\Omega^{-1}}{\pi\lambda l_0^{-1}}\cdot\frac{1+\Gamma_a\Gamma_b}{1-\Gamma_a\Gamma_b}\left(\sqrt{1-\left(\frac{1-\Gamma_a\Gamma_b}{1+\Gamma_a\Gamma_b}\right)^2}-1\right)\right]^2\right\}^{1/2};$$

$$(\omega/\pi\Omega)[(1-\Gamma_a\Gamma_b)/(1+\Gamma_a\Gamma_b)]\left[1-\sqrt{1-(1-\Gamma_a\Gamma_b)^2/(1+\Gamma_a\Gamma_b)^2}\right] \leqslant \lambda l_0^{-1} < (\omega/\pi\Omega).$$

It follows from this that the excitation threshold decreases with increase of zone number (i.e., with increase in Ω). Thus in this class of system it is absolutely necessary to consider not only the first, but also the higher-order parametric resonances. This fact is a result of the multimode nature of the system, a property which is confirmed by experimental studies on systems with distributed parameters which vary with time [7]. The instability growth rate increases with zone number and for v = 0 its maximum value (in the zone centers) is equal to

$$(\omega/2\pi)\ln\{\Gamma_a\Gamma_b[(1+\lambda l_0^{-1}N\pi)/(1-\lambda l_0^{-1}N\pi)]^2\}.$$

It is interesting to note that only the odd zones exist for a stationary string (v = 0) (see Fig. 3a). If the string moves with even a small velocity, then the number of zones doubles and they all become shifted to the right along the ω/Ω axis (Fig. 3b). The width of the zones depends on the velocity of the string (Fig. 4). When $\sin[(\pi N/2)(1 + v/c)] = 0$ the zones degenerate into straight-line segments, the number of segments corresponding to the zone number N.

The number of pulses excited in a system can vary. It depends on the initial conditions. We can show by means of graphical constructions on the space–time plane (see Fig. 1) that the greatest number of excited pulses is equal to the corresponding zone number.

The approach which we have adopted here to the study of the parametric resonance conditions can also be used for more general cases of boundary movement. If, for example, the boundaries do not move in phase and

$$a(t) = \lambda\sin\Omega t;\; b(t) = l_0 + \lambda\sin(\Omega t + \varphi),$$

then in place of (3.3) we get

$$\left|[\omega\Omega^{-1}N(1-v^2/c^2)-1]/2\lambda l_0^{-1}\sin[(N\pi/2)(1+v/c)+\varphi]\right| < 1;$$

$$\pi|\lambda|l_0^{-1} < \omega\Omega^{-1}(1-|v|/c).$$

The phase difference obviously produces a shift of the zones in parameter space along the v/c axis.

§5. The wave processes in the system we have considered are similar in nature to those which occur in a one-dimensional mechanical system where the properties of the boundaries vary with time. An elastic fixing,

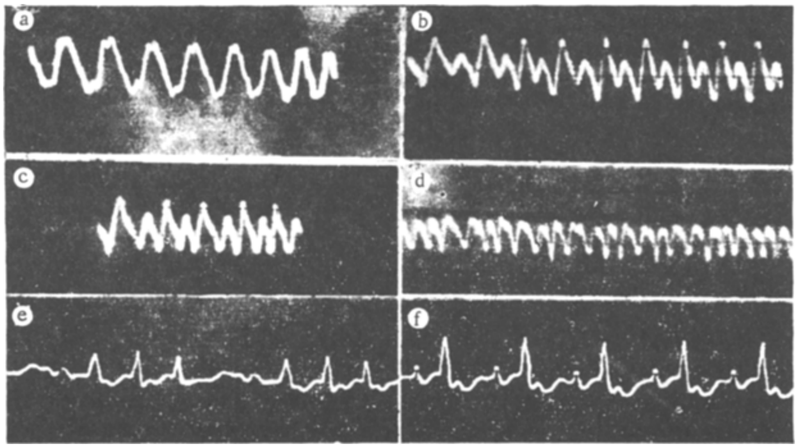

Fig. 6

for example, is equivalent to an extension of the system for harmonic waves. Thus a variation in the rigidity with time will correspond more or less to a variation in the extension and hence the interaction of a wave with this type of fixture can result in compression (or elongation) as in the Doppler effect at a moving boundary.

The parametric excitation of pulse oscillations has been observed experimentally in a distributed mechanical system with a nonstationary boundary (Fig. 5) which took the form of an extended flat rubber band 110 cm in length and 2.5 cm in width. One end of the band was rigidly clamped and the other was held between two steel bending springs. The rigidity of these springs was varied periodically in time by means of a motor carrying a symmetrically placed elliptical cam. The rigidity c thus varied about its average value c_0 according to a more or less harmonic law with a relative modulation depth of $m \geq 0.2$.

With $c = c_0$ and a tension of h = 5 kg, the first 4-5 components in the natural transverse oscillation spectrum of the band were almost equally spaced and at the lowest frequency of $f_0 = 13$ Hz the magnification $Q \geq 40$.

The oscillations were recorded by means of a microphone which was set up near the band. The voltage from the terminals of the microphone was fed to an oscilloscope. The quantity that was observed was thus the time derivative of the transverse displacement at a fixed cross section of the distributed system.

The modulation frequency F of the springs was varied between 10 and 70 Hz. Several zones of parametric instability were found in this range. The oscillations excited in the odd zones ($F = f_0$ and $F = 3f_0$) were almost sinusoidal (Fig. 6a), while those in the even zones ($F = 2f_0$ and $F = 4f_0$) were of the pulse type (Fig. 6b-d). The exact shape of the pulses depended very much on where the microphone was placed along the band. Unipolar pulses were observed near the fixed end (Fig. 6c) and dipolar pulses in the middle (Fig. 6d).

Quite different pulse excitation effects were observed at the edges of the instability zones. The pulses either appeared with every other one missing (Fig. 6e) or occurred in groups (Fig. 6f). This can probably be explained by the nonlinear nature of the system.

We might note, in conclusion, that the parametric pulse excitation effects observed in this system have much in common with the similar effects in electrodynamic distributed systems where the distributed [7] or lumped [8] parameters vary with time.

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LOW-INERTIA PYROELECTRIC DETECTORS FOR RECORDING RADIATION OVER THE 40-1100-nm RANGE

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Pyroelectric radiation detectors which make use of the abrupt temperature dependence of the spontaneous polarization in ferroelectrics have a comparatively high sensitivity, a broad spectral response, and a low inertia [1, 2]. Pyrodetectors are usually used to record infrared radiation.

We consider the operation of a longitudinal-type detector which uses a ferroelectric crystal. The polarization vector P is directed along the x axis perpendicular to the electrodes and the radiation is absorbed by one of the electrodes. The pyroelectric current produced in any element $\Delta x \Delta y \Delta z$ of the crystal is determined by the time rate of change of the polarization $dq/dt = \Delta y \Delta z dP/dt$, and the average current in the crystal is proportional to the change in the average temperature

$$\frac{dq}{dt} = \frac{A}{d}\int_0^d \frac{dP}{dT}\frac{dT}{dt}dx; \quad \frac{dP}{dT} = \gamma; \quad \frac{1}{d}\int_0^d \frac{dT}{dt}dx = \frac{d\bar{T}}{dt}; \quad \frac{dq}{dt} = A\gamma\frac{d\bar{T}}{dt},$$

where A is the area of the crystal surface on which the radiation is incident, d is the thickness of the crystal in the direction of propagation of the thermal wave, and $\gamma = dP/dt$ is the pyroelectric coefficient – a constant over some temperature range below the Curie temperature.

If we neglect thermal losses in the crystal, we can write the thermal balance equation in the form

$$cd d\bar{T}/dt = a dE/dt,$$

where c is the thermal capacity of unit volume of the crystal, E is the radiation energy density, and a is the radiation absorption coefficient of the crystal.

The measuring circuit can be represented as a current generator connected in parallel with the self-capacity of the crystal C_+, the crystal resistance R_+, the circuit capacity C_-, and the load resistance R_-. The resistance of the crystal is usually much greater than the load resistance ($R_+ \sim 10^{10}$–10^{12} $\Omega \cdot$ cm) and can be neglected. The voltage across the load resistance in the case $R_-C_1 \gg \tau^0$ ($C_1 = C_- + C_+$) is

$$U = (Aa\gamma/C_1 dc)E. \tag{1}$$

and when $R_-C_1 \ll \tau_*$ it is

$$U = (Aa\gamma R_-/dc)\cdot dE/dt. \tag{2}$$

The quantities τ^0 and τ_* are the maximum and typical minimum durations of a radiation pulse. Relationships (1) and (2) define two important modes of operation for a pyroelectric detector: the measurement of the energy of a pulse ($R_-C_1 \gg \tau^0$) and the measurement of its power ($R_-C_1 \ll \tau_*$).

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